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BEAM BREAKUP INSTABILITIES IN HIGH CURRENT ELECTRON  
BEAM RACETRACK INDUCT..(U) MISSION RESEARCH CORP

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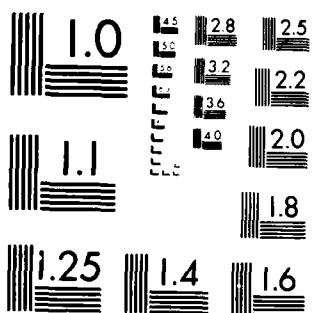
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BEAM BREAKUP INSTABILITIES IN HIGH CURRENT ELECTRON  
BEAM RACETRACK INDUCTION ACCELERATORS

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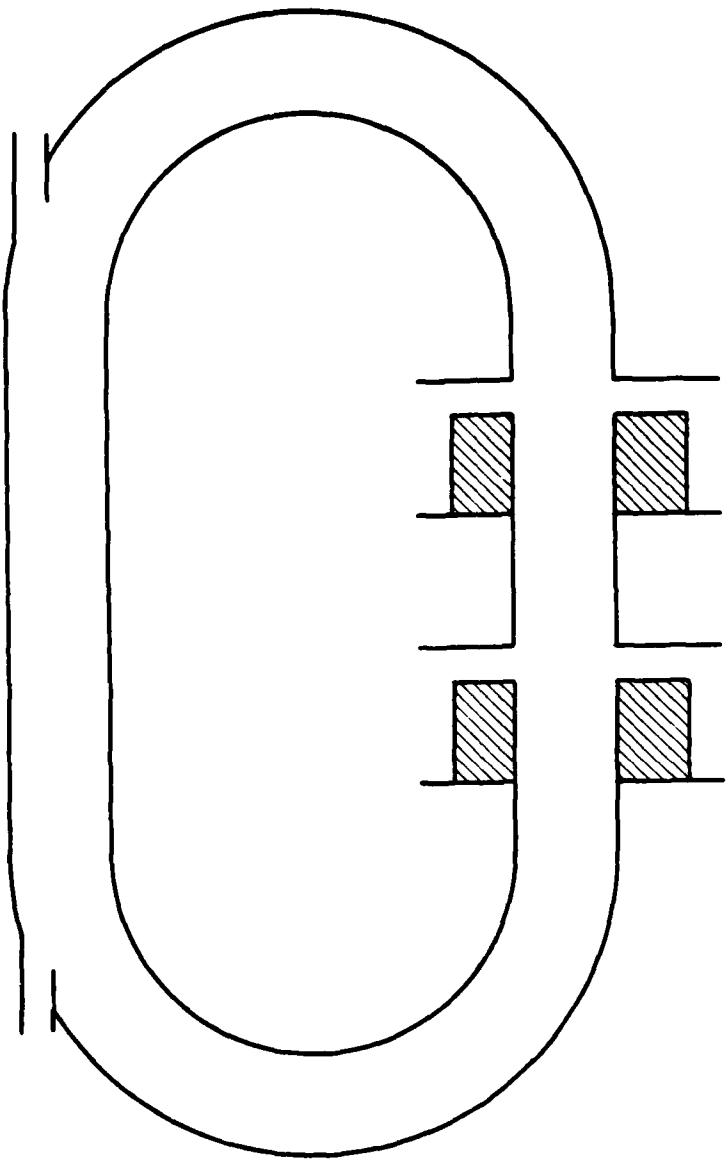
## SUMMARY

Beam breakup and negative mass instability growth rates for a 1 kA, 40 MeV electron beam racetrack induction accelerator are computed. The device is taken to have four acceleration gaps, each with 0.2 MeV applied voltage and 15 ohm transverse impedance; the guide field is 2 kg. We find that the total amplification of the beam breakup mode is limited to five e-foldings provided that the cavity mode quality factor  $Q$  is 6. Thus, the negative mass instability, which grows several times faster, is the dominant consideration. However, we also find that the energy range over which the negative mass instability occurs can be narrowed substantially by reducing the guide field strength after the beam has been accelerated to about 12 MeV. This approach, coupled with beam thermal effects, not considered here, probably is sufficient to limit negative mass growth to acceptable levels in the racetrack accelerator.

## INTRODUCTION

High current racetrack induction accelerators and modified betatrons are a subject of increasing interest as sources of high power electron beams for free electron lasers, flash radiography, and other applications. The racetrack induction accelerator geometry is illustrated schematically in Figure 1. The beam is injected from a conventional pulsed diode beam generator into the drifttube, is progressively accelerated as it repetitively passes one or more induction modules, and then is extracted from the accelerator for its intended use. Extraction may even be unnecessary for microwave applications, because a slow-wave or rippled-magnetic-field cavity can be inserted in a straight section of the drifttube.<sup>1</sup>

Most beam stability studies for high current recirculating devices have dealt with the negative mass and resistive wall instabilities.<sup>2-5</sup> However, experience with linear induction accelerators suggests that beam



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Figure 1. Simplified representation of cyclic induction accelerator with racetrack drift tube, acceleration gaps, and injection and extraction ports.



A

breakup due to interaction with the induction modules and other discontinuities in the drifttube may be significant.<sup>6,7</sup> The beam breakup instability arises from a resonant coupling between beam transverse oscillations and  $m=1$  electromagnetic cavity modes localized to the acceleration gaps, resulting in large lateral displacements of the beam.<sup>8,9</sup> In this paper we present a linear dispersion relation describing both beam breakup and negative mass instabilities, including their possible interaction, and evaluate it for parameters of the proposed racetrack induction accelerator designed by the Naval Research Laboratory.<sup>1</sup>

The NRL device is based on the four module linear induction accelerator developed by the National Bureau of Standards.<sup>10</sup> It is expected to accelerate a 1 kA electron beam from 1 to 40 MeV in fifty cycles. The beam and drifttube radii are 1 and 7 cm, respectively. The principle  $m=1$  resonance of the gaps has a frequency of 880 MHz, an impedance of 15 ohms, and a quality factor ( $Q$ ) of 60. Experience with the ETA linear induction accelerator at Lawrence Livermore National Laboratory indicates that  $Q$  can be greatly reduced, however, and we shall take  $Q=6$  in our numerical work.<sup>11</sup> The NRL design includes a 2 kg axial magnetic field to maintain the beam equilibrium and improve beam stability at low energies. Reducing or eliminating this guide field at higher energies is nonetheless an interesting possibility. The stability analyses below consider both options.

#### DISPERSION RELATION

For simplicity we represent the racetrack accelerator as a torus with a single gap. These two approximations are conservative in that omitting the straight sections of the racetrack and lumping the several gaps into one overestimate negative mass and beam breakup growth, respectively. The desired dispersion relation is

$$(\Omega^2 - \omega_r^2 + F_r/\gamma L + \omega_\theta^2 \gamma^2 x) (\Omega^2 - \omega_z^2 + F_z/\gamma L) \quad (1)$$

$$- (B_\theta/\gamma)^2 \Omega^2 = 0$$

with

$$x = \tilde{v} - \frac{\omega}{\omega_\theta} (\omega v_\theta - \frac{\ell}{R}) / [\Omega^2 + \tilde{v} (\omega^2 - \frac{\ell^2}{R^2})] \quad (2)$$

$$\tilde{v} = (1 + 2\ln a/r_b) v/\gamma^3 \quad (3)$$

Here,  $\Omega = \omega - \ell \omega_\theta$  is the Doppler-shifted wave frequency,  $\ell$  is the toroidal mode number, and  $\omega_\theta = v_\theta/R$  is the toroidal rotation frequency of the beam. (The poloidal rotation frequency is assumed negligible.) The radial and vertical betatron frequencies are

$$\omega_r^2 = (1 - n - n_s r_b^2/a^2) \omega_\theta^2$$

$$\omega_z^2 = (n - n_s r_b^2/a^2) \omega_\theta^2$$

$$n_s \equiv n_b / (2\omega_\theta^2 \gamma^3) \quad (4)$$

with  $n$  the betatron index,  $n_b$  the beam density,  $v = n_b r_b^2/2$  Budker's parameter, and  $\gamma$  the beam energy. The drifttube major radius is  $R$ , the drifttube minor radius is  $a$ , and the beam minor radius is  $r_b$ .  $L=2\pi R$ . The toroidal guide field strength is  $B_\theta$ ; the betatron field strength enters as  $B_z = -\omega_\theta \gamma$ .

In a high current betatron  $\omega_r^2$  and  $\omega_z^2$  can be of either sign. The beam is unstable, however, whenever

$$\omega_B^2 = \omega_z^2 \omega_r^2 / (B_0/\gamma)^2 \quad (5)$$

is negative. To avoid this situation, as well as for simplicity, we take  $n=1/2$ . The energy at which  $\omega_B^2 = 0$  typically is labeled the transition energy,

$$\gamma_{tr} \approx (4 v R^2/a^2)^{1/3} \quad (6)$$

The gap response function  $F$  is defined as<sup>9</sup>

$$\frac{F}{\gamma} = - \frac{Z_\perp}{Q} \frac{\omega_0^3}{\omega^2 + i\omega_0\omega/Q - \omega_0^2} \frac{v}{\gamma} \quad (7)$$

where  $\omega_0$  is the resonant frequency,  $Z_\perp/Q$  is the transverse impedance, and  $Q$  is the quality factor. Setting  $F=0$  in (1) recovers the high current beam negative mass dispersion relation. The negative mass instability occurs for all  $\ell$  over a broad range of energies when  $\gamma > \gamma_{tr}$ . For low  $\ell$  only, one or two instability bands (often overlapping) also may exist when  $\gamma < \gamma_{tr}$ . Three of the six beam modes ( $m=0$  spacecharge,  $m=1$  spacecharge, and  $m=1$  cyclotron;  $m$  is the poloidal mode number) have negative energy and so can couple unstably to the gap fields. Note that coupling in the  $m=0$  spacecharge mode occurs only due to toroidal curvature. Choosing  $R=70$  cm, we find maximum coupling at  $\ell=13$ .

## LARGE B ANALYSIS

For the parameters considered here and toroidal mode numbers in the vicinity of 13, the negative mass instability exists only beyond the transition energy  $\gamma_{tr} \approx 2.9$ . Just above the transition energy the instability is due solely to the interaction between the positive and negative energy  $m=1$  spacecharge modes, while at still higher energies the  $m=0$  spacecharge modes also are involved. This change is readily visible in the negative mass instability growth rate, the dashed curve in Figure 2. Which portion grows faster depends on circumstances.

Although the peak growth rate at lower energy is not readily determined analytically, the higher energy peak is easily shown to be

$$r \approx \frac{\sqrt{3}}{2} [2 \epsilon \omega_0 \omega_B^2 v \gamma^2]^{1/3} \quad (8)$$

Instability ceases for

$$\gamma > [6\sqrt{3} \epsilon R B_0 v (1 + 2 \ln a/r_b)]^{1/2} \quad (9)$$

here about 62.

In the absence of curvature, the beam breakup growth rate also is easily estimated. For  $Q$  not too large,<sup>9</sup>

$$r \approx \omega_0 Q \frac{v Z_1 / Q}{B_0 L} \quad (10)$$

Both  $m=1$  negative energy modes grow at this rate when their frequencies roughly match  $\omega_0$ .

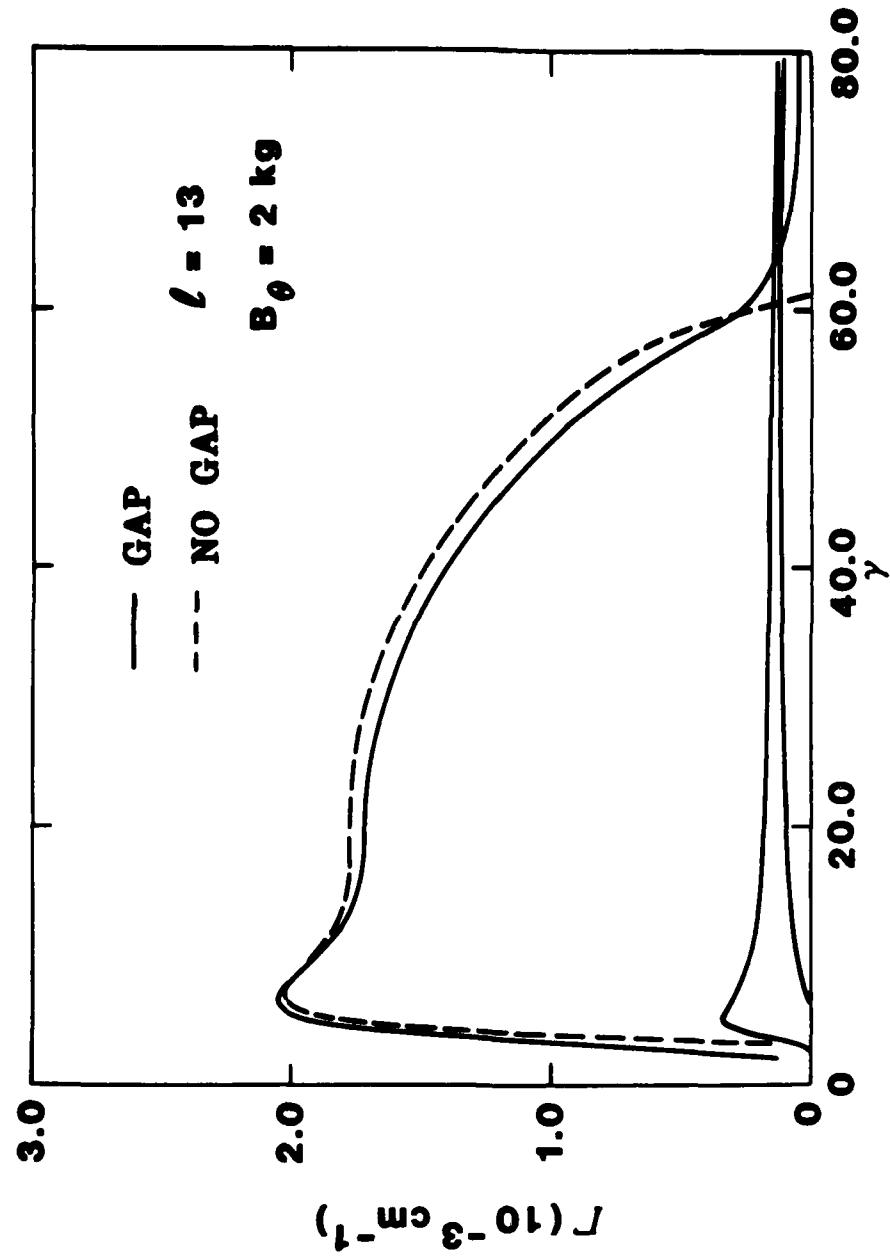


Figure 2. Combined negative mass and beam breakup instability growth rates (solid curves) for  $\ell=13$  and  $B_\theta=2 \text{ kg}$ . Growth of the negative mass instability alone (dashed curve) is included for comparison.

The solid curves in Figure 2 show growth rates of the negative mass and beam breakup instabilities combined. The negative mass results are seen to be only weakly affected by the gap resonance. The  $m=1$  cyclotron and hybrid  $m=0/1$  spacecharge modes have become unstable, however, with a growth rate agreeing with (10) to within a factor of 1.5. These findings are insensitive to small changes in the resonant frequency.

#### SMALL B ANALYSIS

Although modified betatron and racetrack induction accelerator studies usually assume a large toroidal guide field, large  $B_\theta$  is in fact needed to provide a beam equilibrium only for  $\gamma$  small. Reducing or perhaps eliminating  $B_\theta$  after the beam has been accelerated sufficiently has certain advantages for stability, as we see below.

For  $B_\theta=0$ , the negative mass growth rate is approximately

$$\Gamma = \frac{\sqrt{3}}{2} [\epsilon \omega_\theta^3 v^2]^{1/3} \quad (11)$$

This expression exceeds (8) whenever  $B_\theta/\gamma > \omega_\theta$ . However, the corresponding high energy cutoff,

$$\gamma > 3\sqrt{6} \epsilon v (1 + 2 \ln a/r_b) \quad (12)$$

here 27.5, typically is much lower than (9). See the dashed curve in Figure 3.

The beam breakup instability maximum growth rate is again readily estimated, this time giving

$$\Gamma = \omega_0 Q \frac{v Z_1 / Q}{2 \omega_z L \gamma} \quad (13)$$

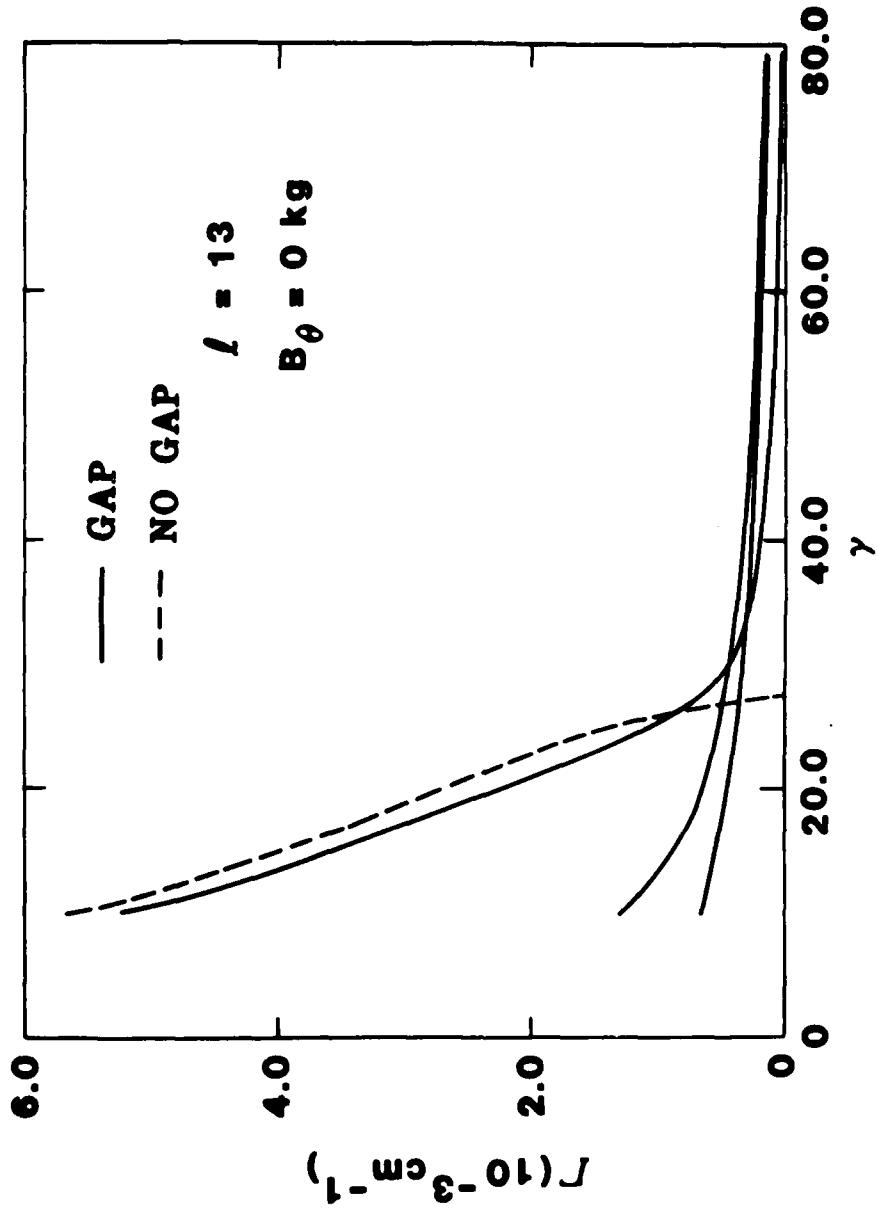


Figure 3. Combined negative mass and beam breakup instability growth rates (solid curves) for  $l=13$  and  $B_\theta=0$  kg. Growth of the negative mass instability alone (dashed curve) is included for comparison.

Equation (13) exceeds (10) for  $B_\theta/\gamma > 2\omega_z$ . The solid curves in Figure 3 show the effects of  $F \neq 0$ . As in Figure 2, the negative mass instability is only slightly modified by the gap; the beam breakup instability is described reasonably well by (13).

A comparison of the two figures suggests that some reduction in total instability growth during acceleration can be achieved by rapidly decreasing the guide field as the beam energy exceeds about 12 MeV.

#### RECOVERING "CONVENTIONAL" NEGATIVE MASS INSTABILITY BEHAVIOR FROM THE HIGH-CURRENT-BETATRON DISPERSION RELATION

The preceding small  $B_\theta$  analysis predicts a high energy cutoff for the negative mass instability in a high current betatron, while standard derivations of negative mass growth, performed for a low current betatron, lead to no such cutoff.<sup>12,13</sup> The source of this apparent discrepancy, namely the failure of the usual approximation

$$1 - \omega_r^2 R^2 / \ell^2 \approx \gamma^{-2} \quad (14)$$

when  $\gamma^4$  becomes large, has been noted previously.<sup>4</sup> To emphasize this point, we here recover the accepted low current growth rate and show that it too exhibits a high energy cutoff, where (14) breaks down. Interestingly, a comparison of the new cutoff with (12), the high current limit, indicates that practical betatron parameters exist for which the negative mass instability does not occur at all at moderate toroidal mode numbers.

We begin by setting  $B_\theta$  (and  $F$ ) to zero in (1).

$$\Omega^2 - \omega_r^2 + \omega_0^2 \gamma^2 x = 0 \quad (15)$$

Next, we drop  $\Omega^2$  as small compared to  $\omega_r^2$ , valid for  $m=0$  spacecharge waves in low current beams, and rearrange terms in the expression for  $x$ .

$$1 = \left( \frac{\omega_\theta}{\omega_r} \right)^2 \gamma^2 - \frac{\dot{v}}{R} \left( \Omega v_\theta - \frac{1}{\gamma^2} \frac{\dot{v}}{R} \right) /$$

$$\left[ \Omega^2 (1 + \tilde{v}) + 2\Omega \tilde{v} \frac{\dot{v}}{R} v_\theta - \tilde{v} \frac{1}{\gamma^2} \frac{\dot{v}^2}{R^2} \right] \quad (16)$$

The conventional negative mass growth rate is obtained immediately by omitting terms linear in  $\Omega$  from (16), an approximation equivalent to invoking (14).

Alternatively, (16) can be solved exactly.

$$2 (1 + \tilde{v}) \Omega = - \tilde{v} \frac{\dot{v}}{R} v_\theta \left( 2 - \frac{\omega_\theta^2}{\omega_r^2} \gamma^2 \right) \pm$$

$$\left[ \tilde{v} \frac{\dot{v}}{R} v_\theta \left( 2 - \frac{\omega_\theta^2}{\omega_r^2} \gamma^2 \right)^2 + 4 (1 + \tilde{v}) \tilde{v} \frac{1}{\gamma^2} \frac{\dot{v}^2}{R^2} \left( 1 - \frac{\omega_\theta^2}{\omega_r^2} \gamma^2 \right) \right]^{1/2} \quad (17)$$

Instability occurs whenever the argument of the square root is negative, approximately

$$\frac{\tilde{v} \gamma^4}{4} \left( \frac{\omega_\theta}{\omega_r} \right)^2 < 1 \quad (18)$$

When (18) is well satisfied, the criterion for (14) to be valid, the desired growth rate is recovered.

$$\Gamma = \tilde{v}^{1/2} \frac{\dot{v}}{R} \frac{\omega_\theta}{\omega_r} \quad (19)$$

This derivation shows clearly the negative mass instability at low currents is associated only with the  $m=0$  spacecharge waves. At higher currents, for which  $\Omega$  and  $\omega_r$  become comparable, the  $m=1$  spacecharge waves also are involved, and the complete quartic equation must be solved to give (11).

For the parameters of Table 1, the low current negative mass instability has a cutoff at  $\gamma = 7.0$ . Since the equilibrium fails below  $\gamma = 10.5$ , we see that the low current instability does not arise. Figure 4, a numerical solution of (1) for 350 A, illustrates growth in the low current limit. The analytical result from (19) at  $\gamma = 10.5$  exceeds the numerical by about 30%.

Rewriting inequality (18) as

$$\gamma > \frac{1}{v} \frac{4(1-n)}{1 + 2\pi n a/r_b} \quad (20)$$

simplifies comparison with (12). Note that (20) predicts a narrow energy bandwidth at high current, while (12) gives the opposite. A window, therefore, exists at moderate currents,

$$v(1 + 2\pi n a/r_b) \sim 0.5-1.0 \quad (21)$$

for which the energy range of the negative mass instability is minimal. For instance, a 750 A, 4.5 MeV electron beam injected into a betatron or racetrack accelerator with dimensions as in Table 1 exhibits no negative mass instability whatsoever for  $z < 6$ . Higher toroidal modes are likely to be stabilized by energy spread<sup>2,4</sup> or nonlinear effects.<sup>14</sup>

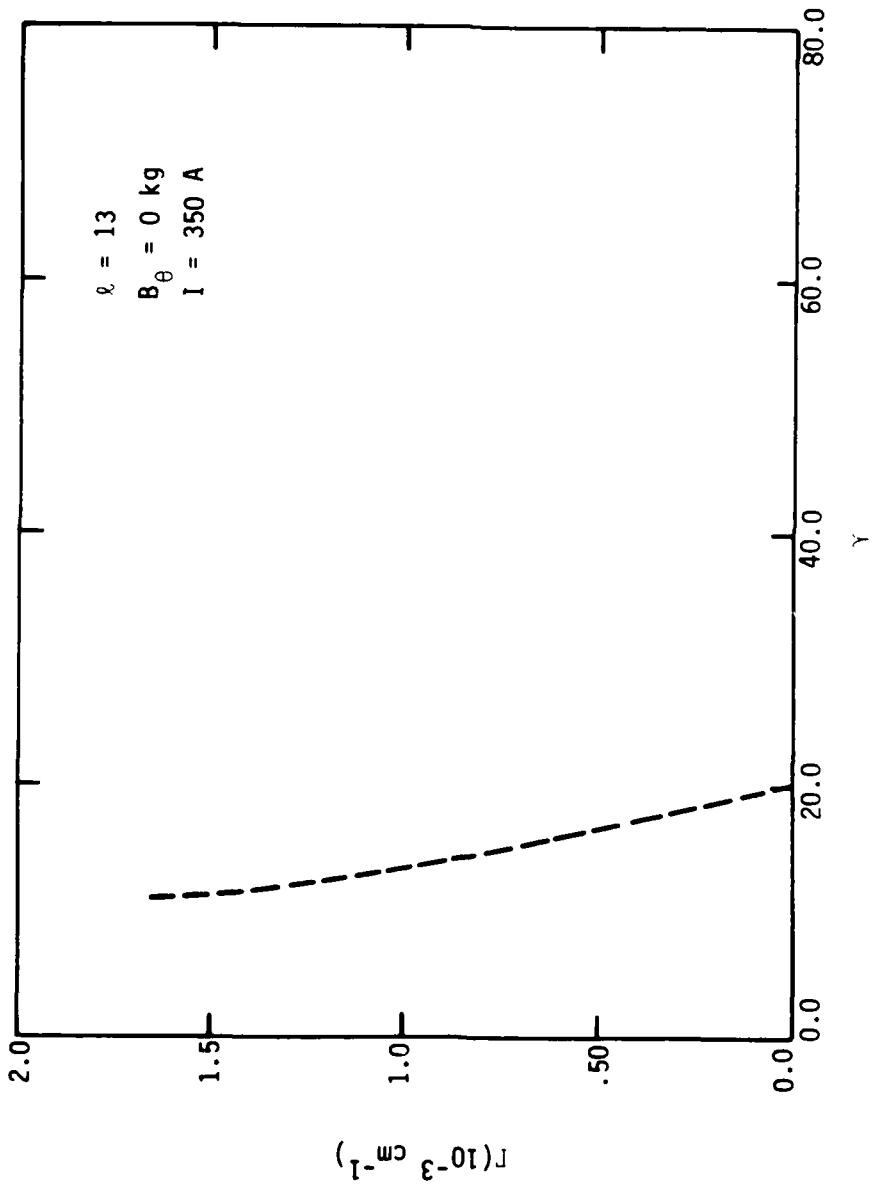


Figure 4. Growth rate of the low current negative mass instability for  $\varrho=13$  and  $B_\theta=0$  kg at  $I=350$  A.

TABLE 1. NOMINAL RACETRACK INDUCTION ACCELERATOR PARAMETERS

Path Lengths	$L = 460 \text{ cm}$
Drifttube Radius	$R = 7 \text{ cm}$
Beam Radius	$a = 1 \text{ cm}$
Guide Field	$B_\theta = 2 \text{ kg} \quad (\omega_c = 1.173 \text{ cm}^{-1})$
Beam Current	$I = 1 \text{ kA} \quad (v = 0.0588)$
Beam Energy	$U = 0.4 - 40 \text{ MeV} \quad (\gamma = 1.5 - 80)$
Number of Revolutions	50
Number of Gaps	$N = 4$
Acceleration per Gap	$\Delta U = 0.2 \text{ MeV} \quad (\Delta\gamma = 0.4)$
Gap Resonant Frequency	880 MHz
Mode Quality Factor	$Q = 60, 6$
Gap Transverse Impedance	15 ohms $(Z_\perp/Q = 0.5)$
Gap Width	$z = 5 \text{ cm}$

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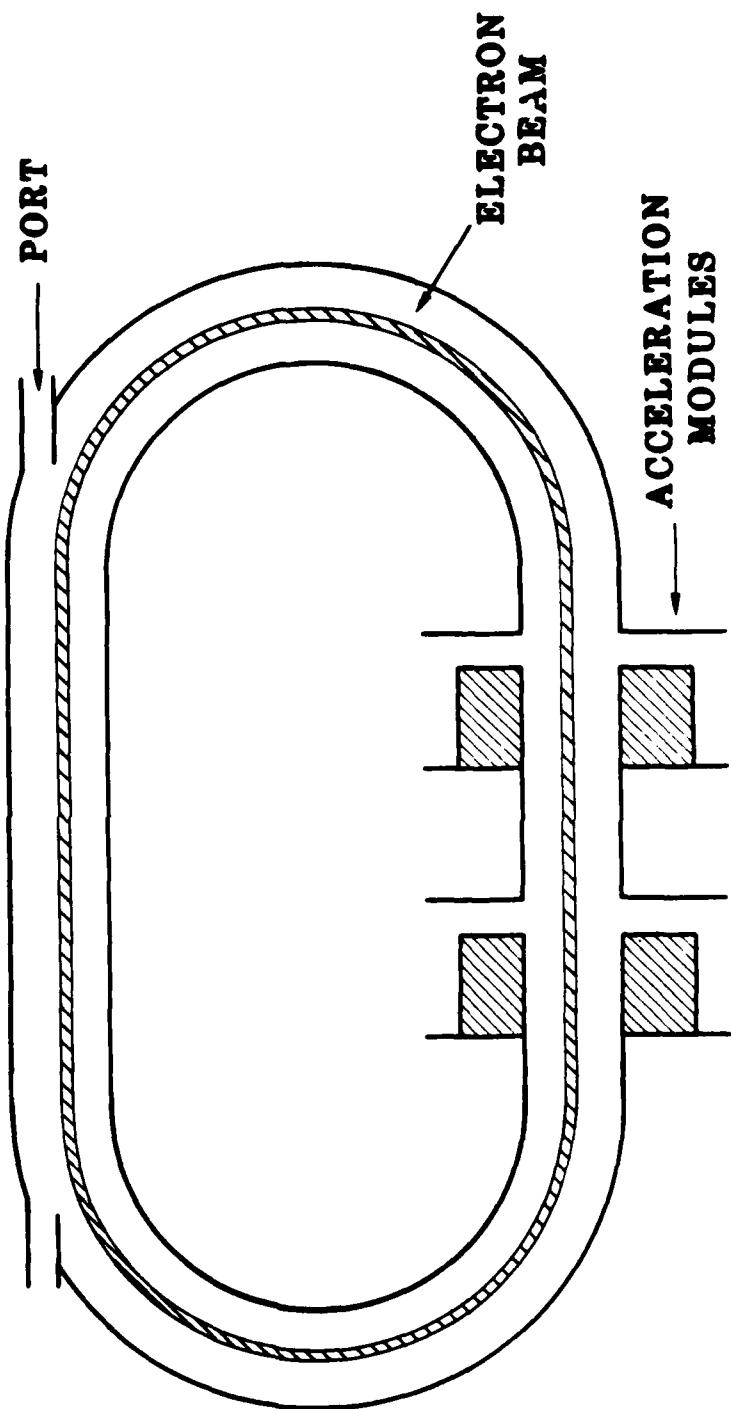
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PARTICLE ACCELERATOR CONFERENCE  
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**RACETRACK INDUCTION ACCELERATOR COMBINES HIGH FIELD  
GRADIENTS, HIGH ELECTRON CURRENTS OF LINEAR INDUCTION  
ACCELERATOR WITH COMPACTNESS OF RECIRCULATING DEVICE.**



BEAM BREAKUP INSTABILITY A CONCERN IN RACETRACK ACCELERATOR.

- ACCELERATION GAPS, OTHER DRIFTTUBE DISCONTINUITIES SUPPORT LOCALIZED ELECTROMAGNETIC RESONANCES
  - INTERACTION OF NEGATIVE ENERGY BEAM WAVES,  $M = 1$  RESONANCES DRIVE LARGE LATERAL BEAM OSCILLATIONS
  - SERIOUS BEAM BREAKUP INSTABILITIES OBSERVED IN LINEAR INDUCTION, OTHER LINEAR ACCELERATORS
  - CURVATURE EFFECTS ON BEAM BREAKUP, INCLUDING COUPLING TO NEGATIVE MASS INSTABILITY, UNKNOWN
- GENERAL DISPERSION RELATION FOR COUPLED BEAM BREAKUP,  
NEGATIVE MASS INSTABILITIES DERIVED BY MRC, SELECTED RESULTS  
PRESENTED HERE.

CALCULATIONS BASED ON NAVAL RESEARCH LABORATORY RACETRACK  
ACCELERATOR DESIGN BY C.W. ROBESON.

BEAM CURRENT ( $\nu$ )	1 kA
BEAM ENERGY ( $\gamma$ )	1-40 MeV
BEAM RADIUS ( $r_b$ )	1 cm
DRIFTTUBE RADIUS ( $a$ )	7 cm
NUMBER OF CYCLES	50
NUMBER OF GAPS	4
GAP VOLTAGE	200 KeV
GAP RESONANCE ( $\omega_0$ )	880 MHz
TRANSVERSE IMPEDANCE ( $Z_{\perp}/Q$ )	15 OHMS
QUALITY FACTOR (Q)	60, 6

PRESENT RESONANCE Q IS 60, BUT CAN BE REDUCED TO 6. LATTER  
VALUE USED IN COMPUTATIONS.

**EXPLORATORY STUDY PERFORMED OF INSTABILITY GROWTH.**

- ANALYTICALLY ESTIMATE NEGATIVE MASS GROWTH WITH NO ACCELERATION GAPS
- ANALYTICALLY ESTIMATE BEAM BREAKUP GROWTH WITH NO TOROIDAL CURVATURE
- NUMERICALLY DETERMINE GROWTH RATES OF COUPLED INSTABILITIES FROM LINEAR DISPERSION RELATION
- VARY AXIAL MAGNETIC FIELD TO MINIMIZE GROWTH

**IMPORTANT EFFECT OF BEAM ENERGY SPREAD NOT INCLUDED, IS KNOWN TO AMELIORATE NEGATIVE MASS INSTABILITY.**

MULTIPLE GAP RACETRACK GEOMETRY SIMPLIFIED TO SINGLE GAP  
 TOROIDAL GEOMETRY TO OBTAIN DISPERSION RELATION:

$$(\Omega^2 - \omega_r^2 + F_r/\gamma L + \omega_\theta^2 \chi) (\Omega^2 - \omega_z^2 + F_z/\gamma L)$$

$$- (B_\theta/\gamma)^2 \Omega^2 = 0$$

$$\chi = \tilde{\nu} \frac{\omega}{v_\theta} \left( \omega v_\theta - \frac{k}{R} \right) / \left[ \Omega^2 + \tilde{\nu} \left( \omega^2 - \frac{k^2}{R^2} \right) \right]$$

$$\tilde{\nu} = (1 + 2 \ln(a/r_b)) \cdot v/\gamma^3 \quad \Omega = \omega - k\omega_\theta$$

$$\frac{F}{\gamma} = - \frac{z_L}{Q} \cdot \frac{\omega_0^3}{\omega^2 + i \omega \omega_0/Q - \omega_0^2} \cdot \frac{v}{\gamma}$$

OMITTING STRAIGHT DRIFTTUBE SECTIONS, LUMPING SEVERAL GAPS  
 INTO ONE LEAD TO SMALL OVERESTIMATES OF GROWTH.

DISPERSION RELATION DESCRIBES THREE PAIRS OF BEAM NORMAL MODES.

•  $M = 1$  CYCLOTRON WAVES

$$\Omega = \pm B_\theta / \gamma$$

•  $M = 1$  SPACECHARGE WAVES

$$\Omega = \pm \omega_B$$

•  $M = 0$  SPACECHARGE WAVES

$$\Omega = \pm \tilde{\nu}^{1/2} l/R\gamma$$

BEAM BREAKUP INSTABILITY OCCURS AT RESONANCE BETWEEN  
 $M = 1$  BEAM, GAP MODES. RESONANCE AT  $l \approx 13$  FOR  $R = 70$  cm.  
( $l, M$  - TOROIDAL, POLOIDAL MODE NUMBERS)

TRANSVERSE OSCILLATION FREQUENCIES STRONGLY AFFECTED BY  
HIGH BEAM CURRENTS.

• RADIAL FREQUENCY

$$\omega_r^2 = (1 - n_s r_b^2/a^2) \omega_\theta^2$$

• VERTICAL FREQUENCY

$$\omega_2^2 = (n - n_s r_b^2/a^2) \omega_\theta^2$$

• BOUNCE FREQUENCY

$$\omega_B^2 = \omega_r^2 \omega_2^2 / (B_\theta / \gamma)^2$$

• SELF-FIELD INDEX

$$n_s = n_b / (2 \omega_\theta^2 \gamma^3)$$

CHOOSE EXTERNAL FIELD INDEX  $n = 1/2$  TO AVOID STRONG  
INSTABILITY FOR  $\omega_B^2 < 0$ .

BEHAVIOR OF NEGATIVE MASS INSTABILITY DEPENDS ON SIGN OF  
 $\omega_r^2, \omega_z^2$ .

- DEFINE TRANSITION ENERGY ( $\omega_r^2, \omega_z^2 = 0$ )

$$\gamma_{tr} \approx (4 \nu R^2/a^2)^{1/3}$$

- FOR Energies  $\gamma < \gamma_{tr}$  ( $\omega_r^2, \omega_z^2 < 0$ )

ONE OR TWO, POSSIBLY OVERLAPPING, NARROW  
INSTABILITY BANDS AT LOW  $\ell$  ONLY

- FOR Energies  $\gamma > \gamma_{tr}$  ( $\omega_r^2, \omega_z^2 > 0$ )  
SINGLE, BROAD INSTABILITY BAND AT ALL  $\ell$

HIGH ENERGY ( $\gamma > \gamma_{tr}$ ) NEGATIVE MASS GROWTH RATE DESCRIBABLE  
ANALYTICALLY.

- BROAD GROWTH PEAK

$$r = \frac{\sqrt{3}}{2} \left[ 2 L \omega_\theta \omega_B^2 \tilde{v} \gamma^2 \right]^{1/3}$$

- INSTABILITY CUTS OFF AT

$$\gamma > \left[ 6 \sqrt{3} L R B_\theta \nu (1 + 2 \ln(a/r_b)) \right]^{1/2}$$

SECOND GROWTH PEAK SOMETIMES OBSERVED JUST ABOVE  $\gamma_{tr}$ .

BEAM BREAKUP GROWTH RATE DESCRIBBABLE ANALYTICALLY IF  
CURVATURE IGNORED.

- GENERAL EXPRESSION ( $\Delta\omega$  - FREQUENCY MISMATCH)

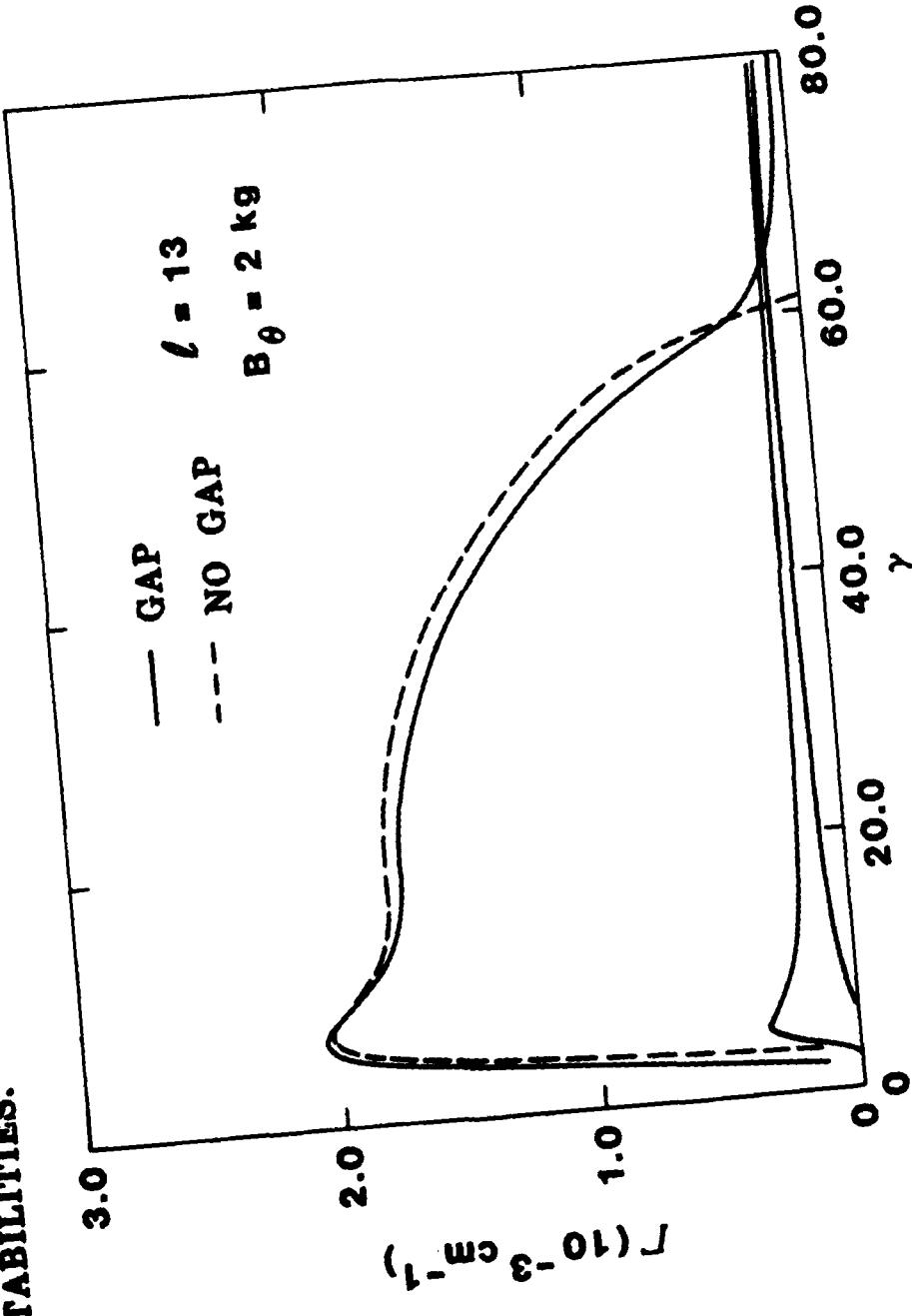
$$\omega - \omega_0 \approx i \frac{\omega_0}{2} \left\{ \left[ \frac{2\nu Z_{\perp}/Q}{B_\theta L} + \left( \frac{1}{2Q} - i \frac{\Delta\omega}{\omega_0} \right)^2 \right]^{1/2} - \left( \frac{1}{2Q} - i \frac{\Delta\omega}{\omega_0} \right) \right\}$$

- LOW Q LIMIT APPROPRIATE HERE

$$f \approx \omega_0 Q \frac{\nu Z_{\perp}/Q}{B_\theta L}$$

( $L = 2\pi R$  - RACETRACK CIRCUMFERENCE)

NUMERICAL SOLUTION OF COMPLETE DISPERSION RELATION SHOWS  
MINIMAL COUPLING BETWEEN NEGATIVE MASS, BEAM BREAKUP  
INSTABILITIES.



**BEAM NORMAL MODES CHANGE SIGNIFICANTLY FOR SMALL GUIDE  
FIELD ( $B_\theta < B_z$ ).**

- $M = 1$  SPACECHARGE (RADIAL)

$$\Omega \approx \pm \omega_r$$

- $M = 1$  SPACECHARGE (VERTICAL)

$$\Omega \approx \pm \omega_z$$

- $M = 0$  SPACECHARGE

$$\Omega \approx \pm \tilde{\nu} 1/2 L/R \gamma$$

**EQUILIBRIUM FAILS FOR  $n_s > n$  WHEN  $B_\theta = 0$ .**

NEGATIVE MASS GROWTH FASTER BUT OVER NARROWER ENERGY  
RANGE FOR  $B_\theta$  SMALL.

APPROXIMATE GROWTH RATE

$$r \approx \frac{\sqrt{3}}{2} \left[ \ell \omega_\theta^3 \tilde{\nu} \gamma^2 \right]^{1/3}$$

INSTABILITY CUTS OFF AT

$$\gamma > 3\sqrt{6} \ell \nu (1+2 \ln(a/r_b))$$

GROWTH RATES WITH, WITHOUT  $B_\theta$  COMPARABLE FOR  $B_\theta/\gamma \sim \omega_\theta$ .

**BEAM BREAKUP GROWTH ESTIMATES (CURVATURE OMITTED)**  
**ALSO STRONGLY MODIFIED FOR SMALL  $B_\theta$ .**

- GENERAL EXPRESSION ( $\Delta\omega$  - FREQUENCY MISMATCH)

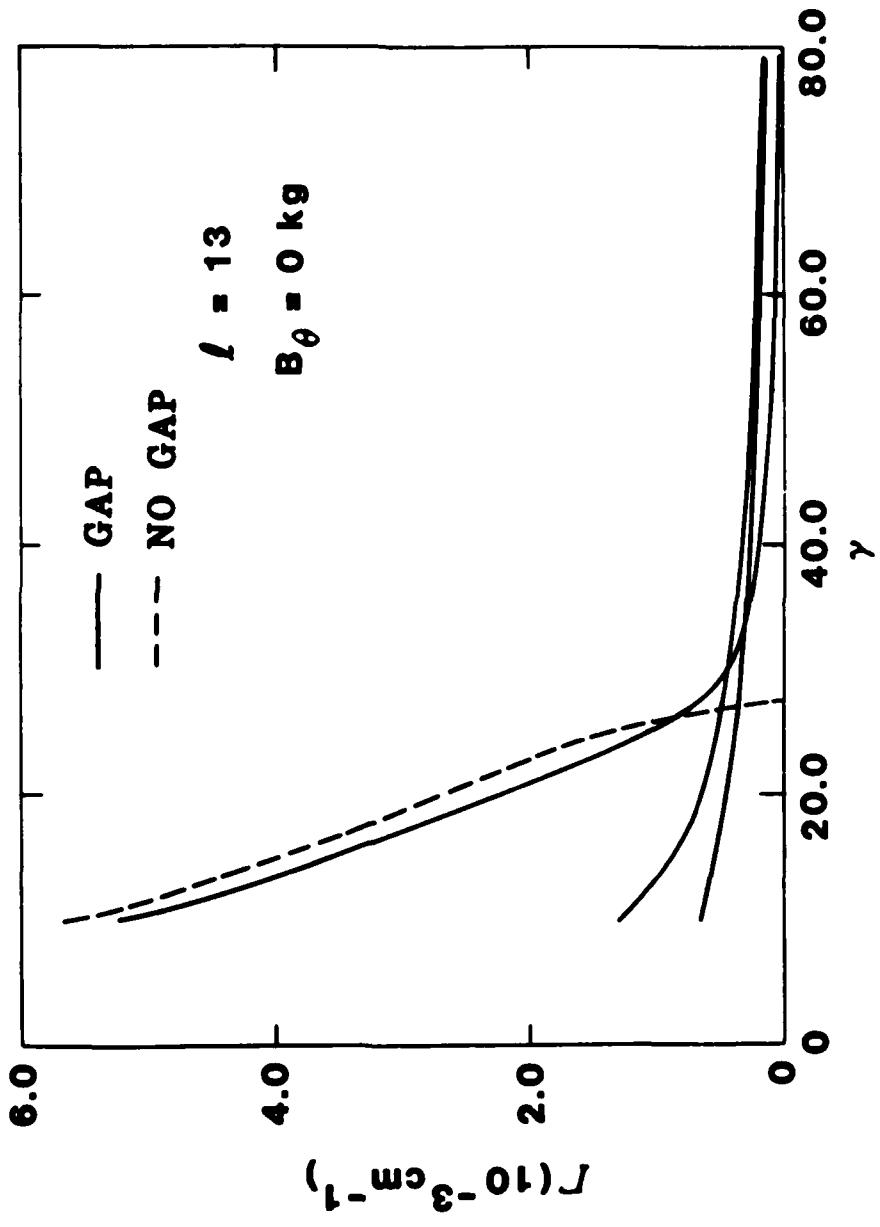
$$\omega - \omega_0 \approx i \frac{\omega_0}{2} \left\{ \left[ \frac{\nu Z_{\perp}/Q}{\omega_z L \gamma} + \left( \frac{1}{2Q} - i \frac{\Delta\omega}{\omega_0} \right)^2 \right]^{1/2} - \left( \frac{1}{2Q} - i \frac{\Delta\omega}{\omega_0} \right) \right\}$$

- LOW Q LIMIT APPROPRIATE HERE

$$\Gamma \approx \omega_0 Q \frac{\nu Z_{\perp}/Q}{2 \omega_z L \gamma}$$

NOTE  $\gamma^{-1}$  DEPENDENCE.

NUMERICAL SOLUTION OF COMPLETE DISPERSION RELATION SHOWS  
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ELECTRON BEAM STABILITY IN HIGH CURRENT RACETRACK  
INDUCTION ACCELERATOR SEEMS PROMISING.

- NO STRONG INTERACTION AMONG INSTABILITIES
  - BEAM BREAKUP GROWTH LIMITED TO FIVE E-FOLDINGS  
OR LESS
  - NEGATIVE MASS GROWTH, 20 E-FOLDINGS IN COLD BEAM  
MODEL, CUT SUBSTANTIALLY BY ELECTRON ENERGY SPREAD
  - RESISTIVE WALL, IMAGE DISPLACEMENT INSTABILITIES (NOT  
CONSIDERED HERE) ARE FAR SLOWER
- POSTER R16 ADDRESSES NONLINEAR EVOLUTION OF NEGATIVE  
MASS INSTABILITY.

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**FRUITFUL DISCUSSIONS WITH NRL HIGH POWER ACCELERATOR  
TEAM ALSO APPRECIATED.**

**ED  
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